

UNIVERSITY OF BALOCHISTAN QUETTA
MA/MSC (ANNUAL) EXAMINATION .2015.

- Attention:-
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Subject:

MATHEMATICS
(Algebra and Complex Analysis)

Paper: I (Prev)

Max: Marks **100**

Time Allowed: **3 Hours**

Note: **Attempt Any 20 Questions from section I in 40 minutes and any Four Questions from Section II , All Questions Carry Equal Marks**

Section I (Time allowed 40 Minutes)

Q.No 1

- I. Define Harmonic function
- II. State the Cauchy-Riemann equations, why it is important.
- III. What do you understand by Residue
- IV. What is analytic function
- V. Represent $-4 + 4i$ in polar coordinate with the principal argument
- VI. Write $(5 - 7i)^2$ in terms of $x + iy$
- VII. State the definition of complex line integral
- VIII. What is Green theorem
- IX. Give the statement of Cauchy integral formula of nth order
- X. Find the Taylor series of the function $\ln z$ at the point $z = 0$
- XI. What is singularity
- XII. What do you understand by Harmonic conjugates.
- XIII. What is a ring?
- XIV. What is a field?
- XV. Define a group
- XVI. Define a subgroup
- XVII. Define a homomorphism
- XVIII. Define a normal sub group
- XIX. Define the order of an element $x \in G$
- XX. If $x, y \in G$ show that xy and yx have the same order
- XXI. Define what is meant by "the subgroup generated by an an element $x \in G$ "
- XXII. List all subgroups of \mathbb{Z}
- XXIII. State the Lagrange's theorem
- XXIV. Define eigenvalues
- XXV. What is a cyclic group?

P.T.O

Section I

(Time Allowed 2:20 hours)

Part -I (Algebra)

Attempt Any two questions from this Part

- Q. No.2 a) Let g_1, g_2 be two elements of a finite group G , then the order of g_1g_2 is same as that g_2g_1
b) Every subgroup of a cyclic group is cyclic

- Q.No.3. a) A subgroup H of group G is normal subgroup of G iff product of two right cosets of H in G is again a right coset of H in G
b) Find the cosets of the additive subgroup $(2\mathbb{Z}, +)$ of the additive group $(\mathbb{Z}, +)$, \mathbb{Z} being set of all integers

- Q.No.4. a) Every finite group G of order n is Isomorphic to a permutation group.
b) Determine whether the given permutations are even or odd

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 5 & 4 & 3 & 6 & 1 & 7 & 9 & 8 \end{pmatrix} \text{ and } \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 3 & 1 & 2 & 6 & 7 & 5 \end{pmatrix}$$

- Q.No. 5. a) Determine whether or not the vector $V = (2, 8, -2, -4)$ is a linear combination of the vectors $u_1 = (1, 2, 0, -3)$, $u_2 = (2, -3, 0, 1)$, $u_3 = (2, -1, -2, 1)$
b) Let S be a non empty finite subset of V . The set of all linear combination of elements in S , denoted by $L(S)$, is a subspace of V containing S and it is the S smallest subspace of V containing S .

Part -II (Complex Analysis)

Attempt Any two Questions from this Part

- Q. No.6 a) State and prove the Cauchy-Riemann equations in polar form
b) Prove that $u = e^{-x}(x \sin y - y \cos y)$ is harmonic, also find harmonic conjugate v and express $f(z) = u + iv$ in terms of z

- Q.No.7. a) State and prove the Cauchy integral formula of order n
b) Evaluate the following

i) $\int_C \frac{zf(z)}{(9-z^2)(z+i)} dz$ where $C = |z| = 2$ ii) Evaluate $\frac{1}{2\pi i} \oint_C \frac{e^{zt}}{(z^2+1)^2} dz$ where $t > 0$ and C is the circle $|z| = 3$

- Q.No.8. a) Expand $\log \frac{1+z}{1-z}$ in a Taylor series and determine the region of convergence
b) State and prove the Laurent's theorem

- Q.No. 9. Evaluate the following

a) $\oint_C \frac{e^{zt}}{z^2(z^2+2z+2)} dz$, where C is the circle $|z| = 3$ b) $\int_0^\infty \frac{dx}{x^4+1}$ c) $\int_{-\infty}^\infty \frac{x \cos mx}{x^2+2x+5} dx$

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Subject:- MATHEMATIC. Paper:- II(Prev:)
Time Allowed :- 3 Hours Analysis (1&II) . Max : Marks : 100

Note :- Attempt Five Questions. in All But Question No. 1- in section –I is compulsory and the time for Section- I is only 40 Minutes. After Expiry of the Time paper should be handed over to the supervisory staff.

SECTION –I (OBJECTIVE PORTION 20 MARKS)

Q.No.1 Short Questions :- (Attempt any 20)

- i. Define Limit of a function .
- ii. Differentiate continuity and limit.
- iii. Distinguish between derivative and integration .
- iv. Define definite integral
- v. Evaluate $\int (3x^2 - 2x) dx$.
- vi. Evaluate $\lim_{x \rightarrow 0} (1 - 1/x)^x$
- vii. Differentiate $y = \cos^5 x$
- viii. Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$
- ix Show that $|a-b| \geq |a| - |b|$
- x. Define increasing and decreasing function
- xi. Define limit of a sequence.
- xii. Differentiate convergence and uniform convergence .
- xiii. What do you know about extreme values .
- xiv. If $a > b$ and $b > c$ then show that $a > c$
- xv. Find the greatest lower bounds of the followings:-
 $E_1 = \{ 1, 2, 3 \} \in \mathbf{Z}$
 $E_2 = \{ 0, \pm 1, \pm 2, \dots \} \in \mathbf{Q}$
- Xvi Define least upper bound.
- xvii. Define bounded variation .
- xviii. Define left hand and right hand limit.
- xix. Explain total variation .
- xx. Define Riemann integral .
- xxi. Define $L(P,f)$ $U(P,f)$
- xxii. Differentiate absolute and relative minima .
- xxiii. Write down any two formula for definite integral .
- xxiv. If $ab = 0$ then show either $a = 0$ or $b = 0$
- xxv. Evaluate limit of $f(x) = \frac{x^3+1}{4x}$ as x Approaches 2 by definition .

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SUBJECT MATHEMATICS PAPER III (Prev)

- i) Differential Equation
- ii) Differential Geometry

Time Allowed – 03 Hours

Max: Marks 100

Note: - Attempt Five Question in All But Question No. 1 is compulsory and the time for section – 1 is only 40 minutes. After Expiry of time Paper, should be handed over to the Examiner.

Section – 1 OBJECTIVE (20 marks)

Q.NO. 1 Attempt any 20 Questions. At least ten from each section

Differential Equation

- 1) Write the expression for Lagrange's linear equation
- 2) What is Clairaut's equation?
- 3) Write expression Bernoulli equation
- 4) Distinguish between ordinary differential equation and partial differential equation
- 5) What is differential equation?
- 6) Define partial differential equation
- 7) Define homogeneous equation.
- 8) Define order of Differential equation.
- 9) Write the formula for Charpit's method.
- 10) What is system of total Differential equations?
- 11) What is linear partial differential equation of 1st order? .
- 12) What is the Cauchy – Euler equation?
- 13) What is an Integrating Factor?
- 14) What is simultaneous equation?
- 15) Define numerical integration.

Differential Geometry

- 16) Explain Differential Geometry
- 17) Explain TNB
- 18) Explain torsion
- 19) Write down Serret Frenet Equations
- 20) Define Helix
- 21) Define Osculating plane
- 22) Write down Spherical Indicatrix
- 23) Define Tensor
- 24) Define Kronecker Delta, with suitable example
- 25) What is contravariant and covariant tensor, give example
- 26) Define Inner Multiplication and give example
- 27) Write the law of transformation for A_i^{ak}
- 28) What is Addition of vectors? and also give example
- 29) Explain Tensor Field
- 30) Explain Quotient law

SECTION II SUBJECTIVE

Time Allowed 2:20

Marks 80

Note: - Attempt any Four Questions at least two from each section. All questions carry Equal marks**Differential Equation**

Q. No. 2 Solve the following Differential Equations.

- i) $2x^2p^2 + 5xyp + 2y^2 = 0$
 ii) $xyp^2 + (x^2 + xy + y^2)p + x^2 + xy = 0$

Q. No. 3 Find the solution of the Miscellaneous Method for the Equation of the Second and Higher Orders.

- i) $yy_2 + y^2_1 = y_1$
 ii) $x \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

Q. No. 4 Solve the following Equations

- i) $\frac{dx}{1} = \frac{dy}{-2} = \frac{dz}{3x^2 \sin(y+2x)}$
 ii) $\frac{dx}{(y+zx)} = \frac{dy}{-(x+yz)} = \frac{dz}{x^2-y^2}$

Q.No.5 Obtained the solution in series by the method of Frobenius

$$9x(1-x)y_2 - 12y_1 + 4y = 0$$

Differential Geometry

Q.No.6

- i) Show Osculating plane of $U = 1$ of curve $r = r(u) = (3au, 3bu^2, cu^3)$ is given by

$$\frac{x}{a} - \frac{y}{b} - \frac{z}{c} = 1$$
- ii) If a tensor B_{lm}^{ijk} is Symmetric Skew Symmetric w.r.t. Indices i and j in one coordinate system, show that it remains Symmetric and Skew Symmetric w.r.t. any coordinate system.

Q.No.7

- i) Write the Law of Transformation for the Tensor and also indicate the rank and order of the tensor $B_{\gamma\beta ml}^{ajak}$
- ii) If A_{lm}^{ijk} and B_{lm}^{ijk} are tensors then prove that their Sum and Difference are also tensor.

Q.8. Define Helix also prove that the curve is helix iff curvature and torsion $\frac{\kappa}{\tau} = \text{constant}$

Q.No.9

- i) Prove Serret Frenet Formulas
 ii) Prove that Necessary and Sufficient Condition for the curve is that $\tau = 0$

University Of Baluchistan, Quetta.

M.A/M.Sc. (Annual) Examination, 2015

Subject:

MATHEMATICS
(Analytical Dynamics and Fluid Dynamics)

Paper-IV (Previous)

Max: Marks 100

Time Allowed: 3 Hours

Note: Attempt any 20 parts from Question I in 40 minutes and Attempt four Questions from Section II in 2:20 hours.

Section I

Q. No. 1.

- i. Define constraints.
- ii. Distinguish between the Body force and Surface force.
- iii. Describe any system that has two degrees of freedom.
- iv. Does the Lagrangian formulation more advantageous than Newtonian formulation.
- v. If Kinetic and potential energy for system are defined as
$$T = \frac{1}{2} m \dot{x}^2 (1 + 4a^2 x^2), V = mgax^2$$
. What will be Lagrangian L for the system.
- vi. A system has only one generalized coordinate i.e x and Lagrangian L for the system is given as $L = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} kx^2$. What will be generalized momentum for given system.
- vii. Define rigid body.
- viii. $F = F(q,p)$ is an integral of motion if $[F,H] = \dots\dots\dots$ where H is Hamiltonian.
- ix. Write the transformation equation for the generating function of Type-3.
- x. What are fundamental Poisson bracket.
- xi. Define central force.
- xii. Under what condition old Hamiltonian is equal to new one.
- xiii. When transformation is said to be canonical.
- xiv. Define Reduced mass.
- xv. By direct method show that given transformation is canonical. $P = \frac{1}{2}(p^2 + q^2), Q = \tan^{-1} \frac{p}{q}$
- xvi. Name any measuring device in which Bernoulli's equation is used.
- xvii. List the assumptions which are made while deriving Bernoulli's equation.
- xviii. Define Energy Head.
- xix. Describe stream function.
- xx. Name the method by which motion of fluid particles may be described by concentrating on a point in fluid.
- xxi. If $\phi = 3xy$ write x and y components of velocity.
- xxii. For $\psi = 2xy$, what will be flow rate between stream lines passing through (2,0) and (2,2).
- xxiii. Velocity vector is everywhere tangent to line in xy -plane along which $\dots\dots\dots$ is constant.
- xxiv. The family of curve given by $\phi = \text{constant}$ and $\psi = \text{constant}$ intersect at angle of $\dots\dots\dots$
- xxv. Distinguish between control volume and control mass system.
- xxvi. What is the difference between real and ideal fluid.
- xxvii. Write unknown velocity component so that continuity equation is satisfied. $u = e^x, v = ?$
- xxviii. Whether the following are true or false
The stream functions exist only in irrotational flow.
- xxix. Velocity potential exists only for irrotational flow.
- xxx. The curl of velocity of particle is equal to angular velocity.

Section II

Note: Attempt four questions from this section, selecting two questions from each part. All questions carry equal marks. (Time Allowed 2:20 hours)

PART - I

Q.No2- a) Show that kinetic energy of system can always be written as sum of three homogenous functions of generalized velocities.

b) A particle of mass M moves in conservative field with cylindrical coordinates (r, θ, z) . Derive Lagrangian and the equation of motion.

Q.No3- a) Solve Brachistochrone problem.

b) Show that $\dot{q}_j = \frac{\partial H}{\partial p_j}$, $\dot{p}_j = -\frac{\partial H}{\partial q_j}$, $\frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$

Q. No 4- a) Verify the following properties of Poisson bracket. $[q_k, p_j] = \delta_{kj}$, $\frac{dG}{dt} = [G, H] + \frac{\partial G}{\partial t}$

b) Prove the following relations for canonical transformation. $\frac{\partial q_j}{\partial Q_k} = \frac{\partial p_k}{\partial P_j}$, $\frac{\partial q_j}{\partial P_k} = -\frac{\partial Q_k}{\partial p_j}$

Q. No 5-a) Prove that $T^2 = \frac{4\pi^2 a^3}{G(m_1 + m_2)}$ where T : period of revolution, a : semi major axis of orbit, m_1 & m_2 are masses of planet and Sun respectively.

b) The eccentricity of the orbit of mercury is about 0.21 and its semi major axis is about 0.39AU. What is the difference between nearest and farthest distance of Mercury from Sun.

PART-II

Q.No6-a) Show that stream line and equipotential line intersect each other at angle of 90° .

b) In a two dimensional flow field for an Incompressible fluid the velocity components are $u = \frac{y^3}{3} + 2x - x^2y$, $v = xy^2 - 2y - \frac{x^3}{3}$. Find an expression for stream function Ψ .

Q.No7- a) Derive the given expression $V_r = \frac{\partial \phi}{\partial r}$, $V_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta}$. Where ϕ is the velocity potential.

b) Does the velocity potential exist for two dimensional Incompressible flow described by $u = x - 4y$, $v = -(y + 4x)$. If so determine its form.

Q.No8-a) Describe the circulation and vorticity in detail.

b) A fluid flow is given by determine whether flow is rotational or irrotational.

$$V_r = \left(1 - \frac{a}{r^2}\right) \cos\theta, V_\theta = -\left(1 + \frac{a}{r^2}\right) \sin\theta$$

Q.No9-a) Derive continuity equation for incompressible steady flow in cylindrical coordinates.

b) Given that $u = xy$, $v = 2yz$ examine whether these velocity components represent two or three dimensional incompressible flow if three dimensional determine the third component.

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Subject:- Mathematics Paper:- V (Prev.)
Time Allowed :- 3 Hours Operation Research Max : Marks : 100

Note :- Attempt Five Questions. in All But Question No. 1- in section –I is compulsory and the time for Section- I is only 40 Minutes. After Expiry of the Time paper should be handed over to the supervisory staff.

SECTION –I (OBJECTIVE PORTION 20 MARKS)

Q.No.1 Write Short Answer :- (Attempt any Ten (10) Questions)

- i) Define operations Research.
- ii) Define Artificial variables.
- iii) Define surplus and slack variables.
- iv) Vogel's Approximation method (VAM) .
- v) Infeasible solution.
- vi) Two phase method.
- vii) Define Degeneracy for simplex method.
- viii) Least cost method.
- ix) What is alternative optima .
- x) Unbounded solution.
- xi) U-V method.
- xii) General transportation algorithm.
- xiii) Optional strategy.
- xiv) Zero sum game .
- xv) Penalty rule for Artificial variables.

SECTION –II(SUBJECTIVE PORTION 80- MARKS) TIME ALLOWED 2:20
Attempt any Four (04) questions.

Q.No. 2 a) Define operations Research , highlight characteristics of OR . also discuss scope of OR in different areas.

Q.No. 3 a) Explain general mathematical formulation of linear programming .
b) Using simplex method, solve the following linear programming problem

$$\begin{aligned} & Z = 4x + 3y \\ \text{i) } & x + 3.5y \leq 9 \\ & 2x + y \leq 8 \\ & x + y \leq 6 \end{aligned} \quad \text{(ii) } x \geq 0 \text{ and } y \geq 0$$

Q.No. 4. a) Define special cases in simplex method approximation i.e. Degeneracy, alternative optima , unbounded solution and Infeasible solution.
b) Show that the M-method will indicate that the following problem has no feasible solution.

$$\begin{aligned} & \text{Maximize } Z = 3x_1 + 2x_2 \\ & \text{Subject to } 2x_1 + x_2 \leq 2 : \\ & \quad 3x_1 + 4x_2 \geq 12 : x_1, x_2 \geq 0 \end{aligned}$$

Q.No. 5.

a) Discuss Big - M method briefly.

b) Solve the following LP model by two phase method.

$$\text{Minimize } Z = 5x_1 - 6x_2 - 7x_3$$

Subject to

$$x_1 + x_2 + x_3 = 5$$

$$x_1 + 5x_2 - x_3 \geq 15$$

$$5x_1 - 6x_2 + 10x_3 \leq 20$$

Where $x_1, x_2, x_3 \geq 0$

a) Find the dual problem of the following primal problem.

$$\text{Minimize } : Z_x = 5x_1 + 20x_2 + 33x_3$$

Subject to

$$(i) \quad 3x_1 - 2x_2 - 4x_3 = 5$$

$$x_1 + 3x_2 + 8x_3 = 45$$

$$(ii) \quad x_1, x_2, x_3 \geq 0$$

Q.No. 7.

a) Write a note on North west corner method.

b) Solve the following by North west corner method.

	Destination				
	10	2	2	20	15
Source	2	7	9	20	20
	4	14	16	18	25
Demand	5	15	15	15	50

Q.No. 8.

a) Solve the following assignment problem.

	I	II	III	IV	V
1	11	17	8	16	20
2	9	7	12	6	15
3	13	16	15	12	16
4	21	24	17	28	26
5	14	10	12	11	13

b) Write a detail note on Hungarian method for solution of assignment problem.

Q.No. 9

Reduce the following game by dominance and find the game value.

Player - B

	I	II	III	IV
Player A	I	3	2	4
	II	3	4	2
	III	4	2	4
	IV	0	4	0

Q.No. 10

Explain Monte Carlo simulation and its algorithm, highlight advantages and disadvantages of Monte Carlo simulation.

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Subject: MATHEMATICS
Paper: Mathematical Methods I &II (VII FINAL)
Time Allowed: 3 Hours Max: Marks 100

Note: Attempt Any 20 Questions from section I in 40 minutes and Attempt any Four Questions from Section II in 2:20 hours.

Section I

Q.No.1.

- I. What is a trigonometric series? A Fourier series?
- II. If $f(x)$ is even then show that $b_n = 0$
- III. Define piece wise continuous function
- IV. Show that $L\{u(t-a)\} = e^{-as} \frac{1}{s}$
- V. Write down the value of $\int_{-L}^L \cos^2 \frac{m\pi x}{L} dx$
- VI. Solve $\frac{\Gamma(3.5)}{\Gamma(2.5)}$
- VII. Solve $\int_0^{\infty} x^2 e^{-x} dx$
- VIII. Solve $\int_0^1 x^4 (1-x)^3 dx$
- IX. Discuss briefly the symmetry of even and odd function.
- X. Write down the Laguerre differential equation
- XI. Prove the symmetric property of Beta function
- XII. Under which conditions gamma and beta functions are convergent.
- XIII. Write down the second order linear differential equations
- XIV. Show that $J_3(x) = \frac{4}{x} J_2(x) - J_1(x)$
- XV. What is unit step function? Why it is important
- XVI. State the formula for the Laplace transform of the nth derivative of a function $f(t)$
- XVII. Does $\tan t$ have Laplace transform? Give a reason for your answer.
- XVIII. Evaluate $\int_{-1}^1 P_4(x) P_5(x) dx$
- XIX. Show that $P_3(x) = \frac{5}{3} x P_2(x) - \frac{2}{3} P_1(x)$
- XX. Verify that given function $\varphi(x) = x + \frac{1}{4}$ is a solution of integral equation $\varphi(x) = x \int_0^{\frac{1}{4}} \varphi(t) dt$
- XXI. Classify the Integral Equations.
- XXII. What is kernel of an Integral Equation? Write down its importance.
- XXIII. What are Fourier sine and cosine transforms
- XXIV. Find the given function as even, odd, neither (with reason) $\begin{cases} \cos^2 x & \text{if } -\pi < x < 0 \\ \sin^2 x & \text{if } 0 < x < \pi \end{cases}$ period is 2π
- XXV. Define Singular integral equations

P.T.O.